5-3 Definite Integrals and the Mean Value Theorem

Learning Objectives:

I can use the properties of definite integrals to evaluate integrals.

I can find the average value of a function.

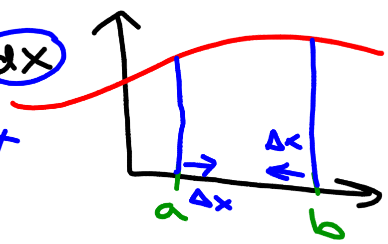
I can apply the Mean Value Theorem (part 2) to find the location where a function takes on the average value.

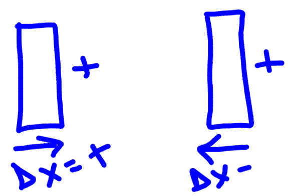
Rules for Definite Integrals

1.) $$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

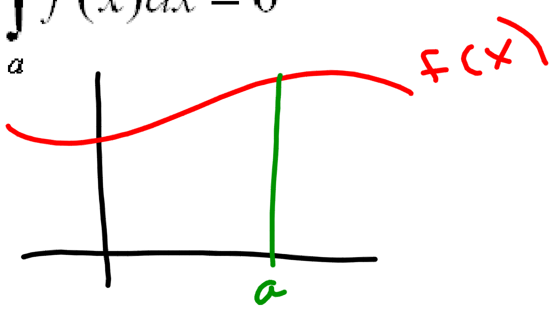
$\int_a^b f(x)dx$

height





2.) $$\int_a^a f(x)dx = 0$$

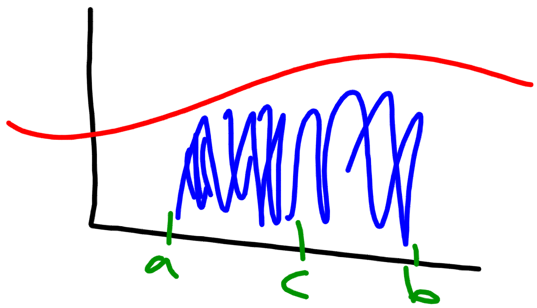


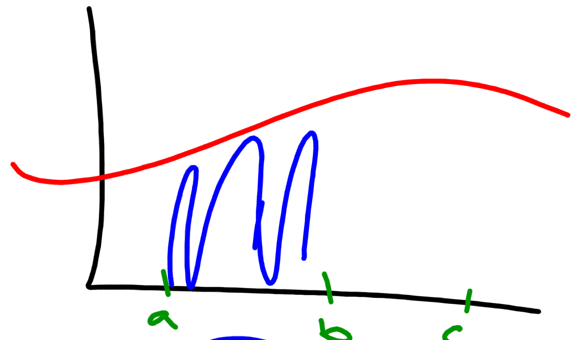
3.) $\int_a^b \boxed{k \cdot f(x)}\, dx = k \cdot \int_a^b f(x)\, dx$

K = Konstant

4.) $\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$

5.) $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

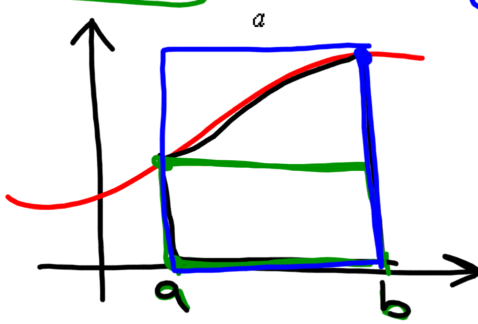




6.) If f_{min} = min value of f(x) on [a,b] and

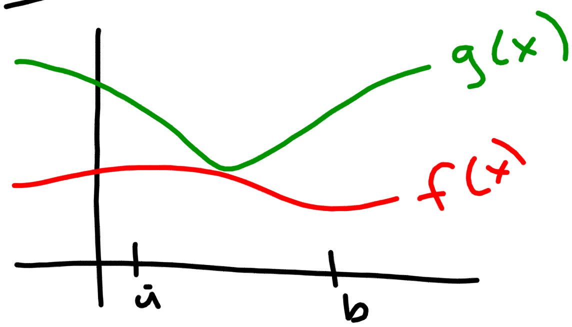
f_{max} = max value of f(x) on [a,b], then

$$\boxed{f_{\min}(b-a)} \le \int_a^b f(x)dx \le \boxed{f_{\max}(b-a)}$$

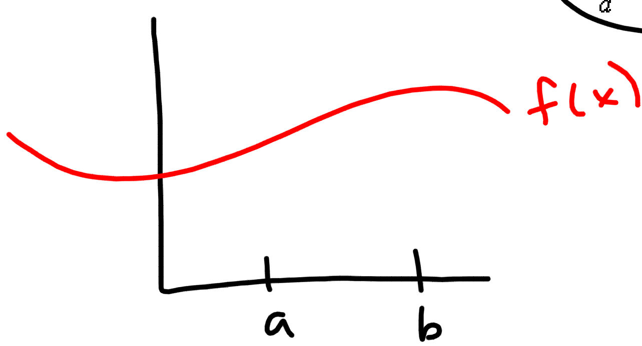


7.) f(x) ≤ g(x) for all x on [a,b], then

$$\int_a^b f(x)dx \le \int_a^b g(x)dx$$



8.) f(x) ≥ 0 for all x on [a,b], then $\int_a^b f(x)dx \ge 0$



Ex1. $\int_{-2}^{1} f(x)dx = 3$, $\int_{1}^{3} f(x)dx = 7$, $\int_{1}^{3} g(x)dx = -3$

Find:

1.) $\int_{-2}^{3} f(x)dx$

2.) $\int_{3}^{1} f(x)dx$

3.) $\int_{1}^{3} 3f(x)dx$

4.) $\int_{1}^{3} [f(x)+g(x)]dx$

5.) $\int_{1}^{3} [2f(x)+5g(x)]dx$

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1. $$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$
$$= 3 + 7 = 10$$

2. $$\int_{3}^{1} f(x)\,dx = -\int_{1}^{3} f(x)\,dx = -7$$

3. $$\int_{1}^{3} 3f(x)\,dx = 3\int_{1}^{3} f(x)\,dx$$
$$= 3 \cdot 7 = 21$$

4. $$\int_{1}^{3} [f(x) + g(x)]\,dx = \int_{1}^{3} f(x)\,dx + \int_{1}^{3} g(x)\,dx$$
$$= 7 + -3 = 4$$

5. $$\int_{1}^{3} [2f(x) + 5g(x)]\,dx = \int_{1}^{3} 2f(x)\,dx + \int_{1}^{3} 5f(x)\,dx$$
$$= 2\int_{1}^{3} f(x)\,dx + 5\int_{1}^{3} f(x)\,dx$$
$$= 2 \cdot 7 + 5 \cdot -3$$
$$= 14 - 15 = -1$$

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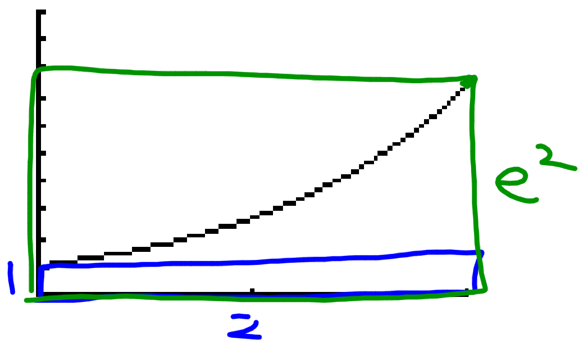
Ex2. Find the upper and lower bounds for $\int_0^2 e^x dx$

$$\boxed{2} < \int_0^2 e^x dx < \boxed{2e^2}$$

14.778

6.389



Average (Mean) Value of a Function

If f(x) is integratable on [a,b], its Average (Mean) Value on [a,b]

$$MV = \frac{1}{b-a}\int_a^b f(x)dx$$

Mean Value Theorem (Part 2) for Definite Integrals

If f(x) is continuous on [a,b], then at some point c in [a,b]

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

a.) Avg value

Ex4. If $f(x) = x^2 + 5x - 7$ on [1,4].

b.) Find a value of ***c*** on [1,4] such that:

a.) $$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

Ans/3

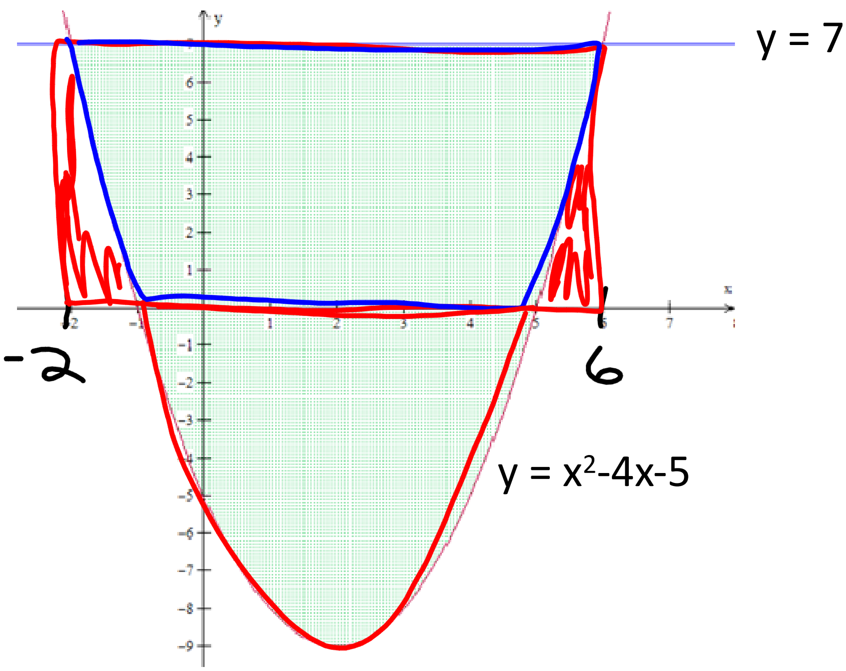
12.5

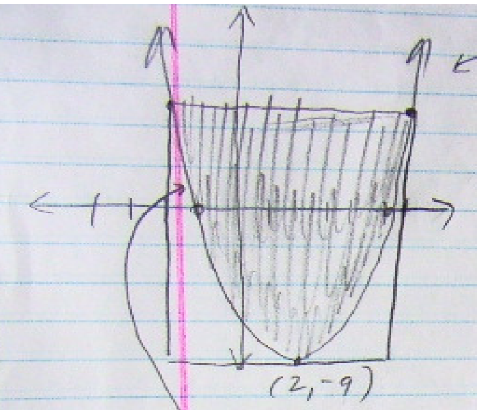
b.) $12.5 = x^2 + 5x - 7$

$0 = x^2 + 5x - 19.5$

$x = 2.574, \; -\cancel{7.}$

Ex5. Find the shaded area.





$$\int_{-2}^{-1} (x^2 - 4x - 5)\,dx = 3.333$$

Top

$A(\square - \int)$

$8 \cdot 7 - 3.333 \cdot 2$

$56 - 6.666$

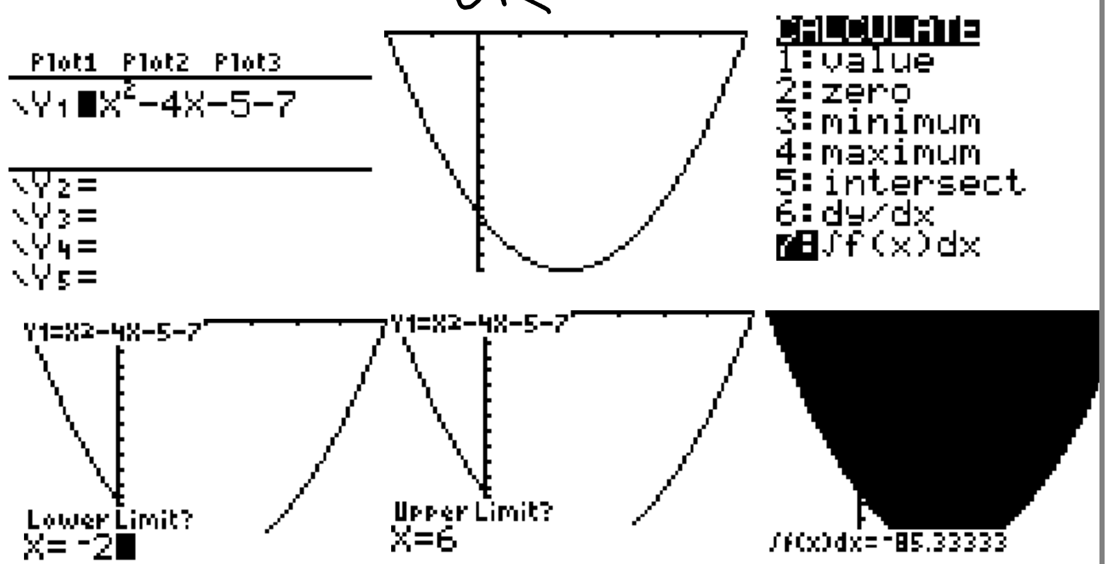
49.334

Bottom

$$+ \left| \int_{-1}^{5} (x^2 - 4x - 5)\,dx \right|$$

$$49.334 + 36 = \boxed{85.334}$$

OR



Homework

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